Teorie- puterea unei matrice pătratice

Definiție. Fie $A \in \mathscr{M}_n(\mathbb{C})$. $A^0 = I_n$, $A^1 = A$, $A^2 = A \cdot A$, $A^n = \underbrace{A \cdot A \cdot \ldots A}_{n \text{ ori}}$, $n \in \mathbb{N}^*$.

Reguli de calcul

1. $A^k \cdot A^p = A^{k+p},\ \forall\ A,B \in \mathscr{M}_n(\mathbb{C}),\ k,p \in \mathbb{N}$

2. $(A^k)^p = A^{kp},\ \forall A \in \mathscr{M}_n(\mathbb{C}),\ k,p \in \mathbb{N}$.

3. $(A+B)^2 = A^2 + AB + BA + B^2,\ \forall A,B \in \mathscr{M}_n(C)$.

3'. *Dacă* $n \geq 2,\ n \in \mathbb{N},\ A,B \in \mathscr{M}_n(\mathbb{C}),\ AB = BA,\ (A+B)^2 = A^2 + 2AB + B^2$.

3''. *Dacă* $n,p \geq 2,\ n,p \in \mathbb{N},\ A,B \in \mathscr{M}_n(\mathbb{C}),\ AB = BA,\ (A+B)^p = \sum_{k=0}^{p} C_p^k A^{p-k} B^k$.

(Formula binomului lui Newton pentru matrice)

4. *Dacă* $m,n,p \geq 2,\ m,n,p \in \mathbb{N},\ A,B \in \mathscr{M}_n(\mathbb{C}),\ AB = BA,\ A^m B^p = B^p A^m$.

Dacă $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathscr{M}_2(\mathbb{C})$, *atunci* $A^2 - (a+d)A + (ad-bc)I_2 = O_2$. *(Relația lui Hamilton − Caylay)*