Varianta 044

SUBIECTUL I

a) $-3i$
b) 1
c) $m = 3$.
d) 5.
e) 27.
f) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$.

SUBIECTUL II

1.
a) $x_1 = 1, x_2 = 4$
b) 1030.
c) $a = 0, b = 1$.
d) $k = 5$.
e) 0.

2.
a) $h'(x) = e^x - 1$.
b) $O(0,0)$ este punctul de minim local.
c) Cum $x_0 = 0$ este punct de minim local atunci $h(x) \geq h(0)$ sau $h(x) \geq 0, \forall x \in \mathbf{R}$.
d) 0.
e) $e - \dfrac{5}{2}$.

SUBIECTUL III

a) $x \circ y = 3xy + 3x + 3y + 2 = 3xy + 3x + 3y + 3 - 1 = 3(y+1)(x+1) - 1, \forall x, y \in \mathbf{R}.$
b) $(x \circ y) \circ z = 3(3xy + 3x + 3y + 2 + 1)(z+1) - 1 = 9(xy + x + y + 1)(z+1) - 1 = 9(x+1)(y+1)(z+1) - 1.$
$x \circ (y \circ z) = 3(x+1)(3yz + 3y + 3z + 2 + 1) - 1 = 9(x+1)(yz + y + z) - 1 = 9(x+1)(y+1)(z+1) - 1.$
Deci $(x \circ y) \circ z = x \circ (y \circ z)$, $\forall x, y, z \in \mathbf{R}$.
c) $a \circ x = x$ devine $3ax + 3a + 3x + 2 = x,$ sau $(x+1)(3a+2) = 0, \forall x \in$.**R.**
Atunci $a = -\dfrac{2}{3}$.
d) $b \circ x = b$ devine $3bx + 3b + 3x + 2 = b$ sau $(b+1)(3x+2) = 0, \forall x \in$ **R .**
Deci $b = -1$.
e) $A = -1$
f)
$1 + 3\log_2 t \cdot \log_3 t + 3\log_2 t + 3\log_3 t + 2 = 0$
$(\log_2 t + 1)(\log_3 t + 1) = 0$
$t = 2^{-1} = \dfrac{1}{2} \in (0, \infty)$ sau $t = 3^{-1} = \dfrac{1}{3} \in (0, \infty)$

Deci $t \in \left\{\frac{1}{2}, \frac{1}{3}\right\}$

g) $a \circ b = 3ab + 3a + 3b + 2 \in$ **Z.**

Fie $k \in$ **Z** astfel incat $3(a+1)(b+1) - 1 = k$

$(a+1)(b+1) = \frac{k+1}{3}$.

Cum $a, b \in$ **Q\Z** alegem $a = \frac{1}{2} \in$ **Q\Z** iar $b = \frac{1}{3} \in$ **Q\Z**

SUBIECTUL IV

a) $f'(x) = 4x^3 - 4$

b) $x^4 - 4x + 1 \geq 2x^2 - 4x$ devine $x^4 - 2x^2 + 1 \geq 0$ sau $(x^2 - 1)^2 \geq 0$. Adevarată $\forall x \in$ **R**.

c) $f'(x) = 0 \Rightarrow (x-1)(x^2 + x + 1) = 0 \Rightarrow x=1$

x	$-\infty$		1		∞
$f'(x)$		-	0	+	
$f(x)$		↘		↗	

Obținem $x_0 = 1$ punct de minim local.

d) $f'(x) = (x-1)(x^2 + x + 1) \geq 0, \forall x \geq 1$.

Deci f este crescătoare pe $[1, \infty)$.

e) Cum $x_0 = 1$ este punct de minim local ,deducem că $f(x) \geq f(1), \forall x \in$ **R** adică $f(x) \geq -2, \forall x \in$ **R.**

f) $\int_1^e \frac{f(x)}{x} dx = \int_1^e \frac{x^4 - 4x + 1}{x} dx = \int_1^e x^3 dx - \int_1^e 4dx + \int_1^e \frac{1}{x} dx = \left.\frac{x^4}{4}\right|_1^e - \left.4x\right|_1^e + \ln|x|\Big|_1^e =$

$= \frac{1}{4}e^4 - 4e + \frac{19}{4}$.

g) $f(x) \geq 2x(x-2), \forall x \in$ **R,** atunci pentru $x > 2$ obținem $\frac{x-2}{f(x)} \leq \frac{1}{2x}$.

Atunci $\int_e^{e^2} \frac{x-2}{f(x)} dx \leq \int_e^{e^2} \frac{1}{2x} dx$.

Dar $\int_e^{e^2} \frac{1}{2x} dx = \frac{1}{2}$. Deci $\int_e^{e^2} \frac{x-2}{f(x)} dx \leq \frac{1}{2}$.