Varianta 5

SUBIECTUL I

a) $M(4,3)$.
b) $m=-1, n=0$.
c) $(1+i)^2=2i$.
d) $\cos^2\frac{\pi}{4}=\frac{1}{2}$.
e) $P=4+6=10$.
f) $V=a^3=8$.

SUBIECTUL II
1.
a) $f(1)+f(2)+\ldots+f(9)=99$.
b) $e=-3$.
c) $p=\frac{2}{5}$.
d) $C_{10}^2=45$.
e) $x=1$.
2.
a) $f(0)=0$.
b) $f'(x)=5x^4, x\in \mathbf{R}$.
c) $\int_0^1 f(x)dx=\frac{1}{6}$.
d) $f(x)+f(-x)=0$.
e) $\lim_{x\to\infty}\left(\frac{1}{5x^6}\cdot\frac{x^6}{6}\right)=\frac{1}{30}$.

SUBIECTUL III

a) $f=X^6+3X^5+6X^4+7X^3+6X^2+3X+1\Rightarrow$ coeficientul cerut este 1.
b) $f(1)+f(-1)=28$.
c) $x^2+x+1=0\Rightarrow x_1=\frac{-1+i\sqrt{3}}{2}$ şi $x_2=\frac{-1-i\sqrt{3}}{2}$.
d) $28=f(1)+f(-1)=2(a+c+e+h)\Rightarrow a+c+e+h=14$.
e) $26=f(1)-f(-1)=2(b+d+g)\Rightarrow b+d+g=13$.

f) Din punctul **c)** $\Rightarrow \omega = \dfrac{-1+i\sqrt{3}}{2}$ este soluție a ecuației $x^2+x+1=0 \Rightarrow f(\omega)=0$.

g) Din **f)**: $x_1 = x_2 = x_3 = \dfrac{-1+i\sqrt{3}}{2}$ şi $x_4 = x_5 = x_6 = \dfrac{-1-i\sqrt{3}}{2} \Rightarrow$

$x_1 + x_2 + \ldots + x_6 = -3$.

SUBIECTUL IV

a) $x_1 = f(1) = \dfrac{1}{3}$.

b) Se obține prin calcul direct.

c) $f'(x) = \dfrac{1}{(2x+1)^2} - \dfrac{1}{(2x-1)^2}, x \ge 1$.

d) $\displaystyle\int_1^2 f(x)dx = \int_1^2 \frac{1}{2}\left(\frac{1}{2x-1} - \frac{1}{2x+1}\right)dx = \frac{1}{4}\ln\left(\frac{2x-1}{2x+1}\right)\Bigg|_1^2 = \frac{1}{4}\ln\frac{9}{5}$.

e) $x_n = \dfrac{1}{2}\left(1 - \dfrac{1}{3} + \dfrac{1}{3} - \dfrac{1}{5} + \ldots + \dfrac{1}{2n-1} - \dfrac{1}{2n+1}\right) = \dfrac{1}{2}\left(1 - \dfrac{1}{2n+1}\right), \forall n \in \mathbf{N}^*$.

f) $\displaystyle\lim_{n\to\infty}(2x_n)^{n+1} = \lim_{n\to\infty}\left(1 - \frac{1}{2n+1}\right)^{n+1} = e^{\lim\limits_{n\to\infty}\frac{-n-1}{2n+1}} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$.

g) $\displaystyle\lim_{n\to\infty}(n+2007)\cdot\ln\left(4n\cdot\left(\frac{1}{2} - x_n\right)\right) = \lim_{n\to\infty}(n+2007)\cdot\ln\left(\frac{4n}{4n+2}\right) =$

$\displaystyle\lim_{n\to\infty}\ln\left(\frac{4n}{4n+2}\right)^{n+2007} = \ln e^{-\frac{1}{2}} = -\frac{1}{2}$.