Varianta 087

SUBIECTUL I

a) $\sqrt{13}$.
b) $a=9, b=-2$.
c) $9\sqrt{3}$.
d) $-4-5i$ **.**
e) $a=1$, $b=-9$.
f) $\sqrt{41}$.

SUBIECTUL II

a) 3.
b) $\frac{2}{5}$.
c) $\frac{1}{2}$.
d) 8.
e) Câtul este X^2+X+1.Restul este 0.
2.
a) $-\frac{6}{x^7}$.
b) -6.
c) $x=0$ este asimptotă verticală.
d) $\frac{31}{160}$.
e) 1.

SUBIECTUL III

a) $A+I_2=\begin{pmatrix}3 & -1\\ 9 & -3\end{pmatrix}+\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}4 & -1\\ 9 & -2\end{pmatrix}=B$.

b) $A^2=A\cdot A=\begin{pmatrix}3 & -1\\ 9 & -3\end{pmatrix}\begin{pmatrix}3 & -1\\ 9 & -3\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}=O_2$.

c) $B^2=\begin{pmatrix}7 & -2\\ 18 & -5\end{pmatrix}$.

d) $\frac{4x-1}{9x-2}>\frac{1}{3}$ sau $\frac{4x-1}{9x-2}-\frac{1}{3}>0$ sau $\frac{3x-1}{9x-2}>0$.

Cum $x>\frac{1}{3}$ avem $3x-1>0$.

Cum $3x > 1$ avem $9x > 3 > 2$. Deci $9x - 2 > 0$. Atunci $\frac{3x-1}{9x-2} > 0$, $\forall\ x > \frac{1}{3}$.

e) $(f \circ f)(x) = \frac{7x-2}{18x-5}$.

f) $B^2 = \begin{pmatrix} 7 & -2 \\ 18 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 2\begin{pmatrix} 3 & -1 \\ 9 & -3 \end{pmatrix}$.

Presupunem $B^k = I_2 + kA$, $\forall k \in \mathbf{N}^*$. Demonstrăm $B^{k+1} = I_2 + (k+1)A$, $\forall k \in \mathbf{N}^*$.

$B^{k+1} = B^k \cdot B = (I_2 + kA) \cdot B = (I_2 + kA) \cdot (A + I_2) = A + I_2 + kA^2 + kA =$

$= I_2 + (k+1)A + O_2$, deoarece din **b)**, $A^2 = O_2$.

Am obținut $B^{k+1} = I_2 + (k+1)A$, adevărată $\forall k \in \mathbf{N}^*$.

Atunci $B^n = I_2 + nA$, $\forall n \in \mathbf{N}^*$.

g)Din **e)**, $(f \circ f)(x) = \frac{7x-2}{18x-5} = \frac{(3 \cdot 2 + 1)x - 2}{9 \cdot 2x + 1 - 3 \cdot 2}$.

Presupunem $(\underbrace{f \circ f \circ \ldots \circ f}_{de\ k\ ori\ f})(x) = \frac{(3k+1)x - k}{9kx + 1 - 3k}$. Demonstrăm

$(\underbrace{f \circ f \circ \ldots \circ f}_{de\ k+1\ ori\ f})(x) = \frac{(3k+4)x - k - 1}{9(k+1)x + 1 - 3(k+1)}$.

$$(\underbrace{f \circ f \circ \ldots \circ f}_{de\ k+1\ ori\ f})(x) = (\underbrace{f \circ f \circ \ldots \circ f}_{de\ k\ ori\ f})(f(x)) = \frac{(3k+1) \cdot \frac{4k-1}{9k-2} - k}{9k \cdot \frac{4x-1}{9x-2} + 1 - 3k} = \frac{x(3k+4) - k - 1}{9k(x+1) + 1 - 3(k+1)},$$

deci $(\underbrace{f \circ f \circ \ldots \circ f}_{de\ n\ ori\ f})(x) = \frac{(3n+1)x - n}{9nx + 1 - 3n}$, $\forall n \in \mathbf{N}^*$, $\forall x > \frac{1}{3}$.

SUBIECTUL IV

a) $u'(x) = -\frac{1}{(x-1)^2} - \frac{1}{(x-2)^2} - \frac{1}{(x-3)^2}$.

b) $f(x) \cdot u(x) = (x-2)(x-3) + (x-1)(x-3) + (x-1)(x-2). = g(x)$;

$g(x) = f'(x) = (x-2)(x-3) + (x-1)(x-3) + (x-1)(x-2)$.

c)Din **a)**, $u'(x) = -\left[\frac{1}{(x-1)^2} + \frac{1}{(x-2)^2} + \frac{1}{(x-3)}\right] < 0$, $\forall x \in A$.

d) Din **b)**, avem $u(x) = \frac{g(x)}{f(x)}$. Atunci

$$u'(x) = \frac{g'(x) \cdot f(x) - g(x) \cdot f'(x)}{f^2(x)} = \frac{f(x) \cdot h(x) - g^2(x)}{f^2(x)}, \forall x \in A.$$

e) $\lim\limits_{x\to\infty} u(x) = \lim\limits_{x\to\infty}\left(\frac{1}{x-1}+\frac{1}{x-2}+\frac{1}{x-3}\right)=0$, deci $y=0$ este asimptota orizontala spre $+\infty$

f) $\int_4^5 u(x)dx = \left(\frac{1}{x-1}+\frac{1}{x-2}+\frac{1}{x-3}\right)dx == \left(\ln\left|(x-1)(x-2)(x-3)\right|\right)\Big|_4^5 = \ln\frac{24}{6} = \ln 4 = 2\ln 2$.

g)Din **c),** $u'(x)<0, \forall x\in A$, deci $\frac{f(x)\cdot h(x)-g^2(x)}{f^2(x)}$<0 .Rezultă $f(x)h(x)-g^2(x)<0$ sau $g^2(x)>h(x)\cdot f(x), \forall x\in \mathbf{R}$.